Thinking About Place Value in Grade 2

Introduction: Two Primary Themes in Grade 2

Students in Grade 2 focus intensively on addition and subtraction. While there are many facets to this work (e.g., 2.MD.4–6), all of them important, two primary themes characterize the major part of it:

1. Using addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions. (From standard 2.OA.1. See also Table 1 on page 88 of the Standards.)

2. Using place value understanding and properties of operations to add and subtract. (Cluster heading for standards 2.OA.5–9.)

In a sense, #1 is "applied math" while #2 is "pure math." Both are important, and indeed both can work together. Word problems reinforce what addition and subtract mean, both in the abstract as well as in terms of what kinds of quantitative relationships these operations can model (such as when a quantity in a word problem increases). In middle school, students will draw on this understanding of addition and subtraction when they set up algebra equations to solve word problems. Real-life situations also provide context and empirical support for the mathematical properties of addition (commutativity and associativity, which combine to make the so-called "any which way rule") and for the mathematical relationship between addition and subtraction (subtraction is an unknown-addend problem). Much of this happens initially in Grade 1 (1.OA.1–6), but it gets reinforced and strengthened during Grade 2, particularly because Grade 2 focuses on the harder addition and subtraction situation types (see Table 2 on page 9 of the draft *Progression* for Operations and Algebraic Thinking, http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf).

But it isn't enough to understand what addition and subtraction mean and how they can be applied to solve word problems. Students in Grade 2 must also learn general and efficient methods for expressing the sum or difference of two multi-digit numbers as another multi-digit number (working within 1,000 at this grade; note that fluency is expected within 100, and single-digit sums must be known from memory by year's end). This effort is what #2 and standards 2.OA.5–9 are all about.

Understanding Place Value

Students aren't likely to meet the standards for adding and subtracting within 1,000 without a solid understanding of place value to 1,000.[*] Understanding place value is what standards **2.NBT.1–4** are all about.

There are two main advantages to the place value system. One is that it allows us to express very large numbers compactly. In just a few seconds, I can type a number larger than the number of atoms in the entire Milky Way galaxy:

9872459087300872309870932871098327409187439817509348918729870239813121.

There isn't enough paper on earth to write this number using hash marks—but it fits on a single line using place value notation.

The second advantage of the place value system is that it allows for efficient computation of sums, differences, products, and quotients. If a minister tells the Roman emperor that there are CLXXVII legions in Gaul, and that each legion needs MMCCXCIV loaves of bread, then the minister had better hope that the emperor doesn't demand a fast answer to the question of how many loaves of bread are needed altogether. But in the place value system, there is a pretty fast algorithm for 2294×177.

How does the place value system work? The fundamental components of the system are:

- *Base-ten units*: Ones, tens, hundreds, thousands, and so on. Students have to get a feel for the sizes of these units (up to thousands in this grade).

- *Flexible bundling and unbundling*: Base-ten units can be broken down and built back up in different ways. By definition, ten ones make a larger unit called "a ten." By definition, ten tens make a larger unit called "a hundred." And by definition, ten hundreds make a larger unit called "a thousand." Therefore it is equally true that a hundred ones make a hundred; a thousand ones make a thousand; and a hundred tens also make a thousand. Bundling and unbundling are central ideas in developing computation algorithms. Rods, flats, and cubes are good for illustrating the ways to bundle and unbundle base-ten units.

- *Positional notation*: According to convention, the location of each digit in a multi-digit number corresponds to a particular base-ten unit. Also, the digit itself tells how many copies of that unit are in the number.[†] So, 908 is 9 hundreds and 8 ones.

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[*] To say that students won't meet standard Z without meeting standard Y is not to say that they must demonstrate that they have met standard Y *before* they go on to learn the math that will allow them to meet standard Z. The curriculum might be structured so that students gradually work simultaneously toward meeting both Y and Z.
[†] By using the word "copies," we mean to describe an essentially multiplicative notion in terms appropriate to grade 2.

Understanding place value means understanding *all three of these things in connection with one another*. So for example, working with rods, flats, and cubes alone won't by itself teach place value, because these manipulatives do not have any connection to positional notation. The place value system belongs to the art of writing. Working with place value requires *writing numbers* with an understanding of how they name quantities.

Understanding place value is having number sense of the base ten units, understanding how these units are bundled and unbundled at will, and connecting this understanding to the positional notation system. All of this becomes a target of operational thinking as students develop computational algorithms.

Assessing Conceptual Understanding of Place Value

The Common Core State Standards present a balanced approach to mathematics that stresses equally the goals of conceptual understanding, fluency, and application. Many in the field welcomed this development in math education while expressing uncertainty as to how conceptual understanding in particular could be assessed. On the next page are some short questions that are meant to suggest ways in which teachers can assess place value understanding among their students. Questions like these could be part of an in-class quiz or unit test.

Following the questions is an annotated version of the page that identifies some of issues highlighted by each group of questions. Finally, on the last page is a released item for place value from a statewide test with some commentary.

Make true equations. Write one number in every space. Draw a picture if it helps.

1) 1 hundred + 4 tens = _____

2) 4 tens + 1 hundred = _____

3) 14 tens = __10__ tens + _____ tens

    = _____ hundred + _4_ tens

    = _____

4) 7 ones + 5 hundreds = _____

5) 8 hundreds = _____

6) 106 = __1_ hundred + _____tens + _____ones

7) 106 = _____tens + _____ones

8) 106 = _____ones

9) 90 + 300 + 4 = _____

Are these comparisons true or false?

10) 2 hundreds + 3 ones > 5 tens + 9 ones   _____

11) 9 tens + 2 hundreds + 4 ones < 924   _____

12) 456 < 5 hundreds   _____

This task relates to the first cluster in 2.NBT: "Understand place value." Within that cluster, it relates to 2.NBT.1, 2.NBT.3, and 2.NBT.4.

Make true equations. Write one number in every space. Draw a picture if it helps.

1) 1 hundred + 4 tens = _____

2) 4 tens + 1 hundred = _____

These two problems have the same answer, 140. This emphasizes that order doesn't matter in addition – yet order is everything in positional notation! In the second problem, you must really think to encode the quantity in positional notation.

3) 14 tens = __10__ tens + _____ tens

   = _____ hundred + __4__ tens

   = _____

In these three problems, the base-ten units in 140 are bundled in different ways. In the first line, "tens" are thought of as units: 14 *things* = 10 *things* + 4 *things*.

4) 7 ones + 5 hundreds = _____

By scrambling the usual order, the problem requires students to link *the values of the parts* with *the order of the digits* in the positional system. Also, to encode the quantity, the student will have to think: "no tens," emphasizing the role of 0.

5) 8 hundreds = _____

When the student writes "8-0-0," the zeros should come with a silent "no tens and no ones."

6) 106 = __1__ hundred + _____tens + _____ones

7) 106 = _____tens + _____ones

8) 106 = _____ones

In these three problems, the base-ten units in 106 are bundled in different ways. This is helpful when learning how to subtract in a problem like 106 – 37, for example.

9) 90 + 300 + 4 = _____

If the order is always given "correctly," then all we do is teach students rote strategies without thinking about the size of the units or how to encode them in positional notation.

Are these comparisons true or false?

10) 2 hundreds + 3 ones > 5 tens + 9 ones

   _____

11) 9 tens + 2 hundreds + 4 ones < 924

   _____

12) 456 < 5 hundreds

   _____

At first, comparisons of numbers should hinge on the sizes of the quantities—not a rote "alphabetization" strategy of comparing digits from left to right. These problems invite the mathematical strategy of looking first for the largest base-ten unit on each side.

A more advanced problem might also involve bundling. For example, True or False: 20 tens + 30 ones > 230.

The chart below shows the numbers of visitors at four parks last year.

Park Visitors Last Year

Park	Number
River Park	8346
Oak Park	9103
Lake Park	9088
Green Park	6299

Which park had the most visitors last year?

Ⓐ   River Park

Ⓑ   Oak Park

Ⓒ   Lake Park

Ⓓ   Green Park

The trouble with this task is not that it's multiple choice. There's nothing wrong with the task, really. It's just that you shouldn't rely on a problem like this one to tell you if your students really understand place value.

Students can solve a problem like this by learning a rote strategy of comparing digits from left to right. If you think about it, that's the same strategy you use to alphabetize words. It's kind of interesting that an alphabetization strategy can be used to order numbers in base-ten notation! But such a strategy should come as the endpoint of one's study of place value—a result of one's understanding, rather than a substitute for it.